196. Proposed by L. E. NEWCOMB, Los Gatos, California.

Find the rth term of $\left(x-\frac{1}{x}\right)^n \equiv z^n$ in terms of z.

I. Solution by G. W. GREENWOOD, B. A. (Oxon), Professor of Mathemrtics and Astronomy, McKendree College, Lebanon Ill.

Let $x=\cos\theta+i\sin\theta$, then $z=2i\sin\theta$, and $x^p=\cos p+i\sin p\theta$. This enables us to express any term of $(x-1/x)^n$ in terms of θ, *i. e.*, in terms of z.

II. Solution by F. D. POSEY, A. B., San Mateo, Cal.; G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va., and GRACE M. BAREIS, Bala, Pa.

$$x-1/x=z, \ \therefore x=\frac{z\pm\sqrt{(z^2+4)}}{2}.$$

The rth term of $(x-1/x)^n$ is $\dfrac{(-1)^{r-1}n(n-1)\ldots\ldots(n-r+2)}{(r-1)!}x^{n-r+1}\left(\dfrac{1}{x}\right)^{r-1}$

$$=\frac{(-1)^{r-1}n(n-1)\ldots\ldots(n-r+2)}{(r-1)!}x^{n-2r+2}$$

$$=\frac{(-1)^{r-1}n(n-1)\ldots\ldots(n-r+2)}{(r-1)!}\left(\frac{z\pm\sqrt{(z^2+4)}}{2}\right)^{n-2r+2}.*$$

197. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

Solve $(18)^{4(2-x)}=(54\sqrt{2})^{3x-2}$.

Solution by W. W. LANDIS, Dickinson College, Carlisle, Pa.

Writing the equation in the form (1)..... $18^{4(2-x)}=18^{\frac{3}{2}(3x-2)}$ we find $x=\frac{22}{17}$.†

Also solved by G. W. Greenwood, J. E. Sanders, A. H. Holmes, F. D. Posey, R. A. Wells, G. I. Hopkins, H. R. Higley, G. B. M. Zerr, E. L. Sherwood, Grace M. Bareis, J. Scheffer, L. E. Newcomb.

GEOMETRY.

219A. Proposed by H. F. MacNEISH, A.B., Assistant in Mathematics. University High School, Chicago, Ill.

Draw through a given point a line which shall divide a given quadrilateral into two equivalent parts: (1) when the point lies in a side of the quadrilateral; (2) when the point is without; (3) within the quadrilateral.

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*Dr. Zerr also gives the values

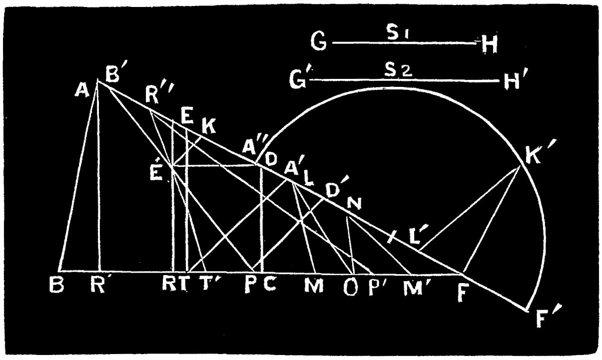
$$(-1)^{r-1}{}_nc_{r-1}(z+1/z-1/z^3+1/z^5-\ldots\ldots)^{n-2r+2},$$

$$(-1)^r{}_nc_{r-1}(1/z-1/z^3+1/z^5-\ldots\ldots)^{n-2r+2}.$$

†Owing to the periodicity of the exponential function a^y may be written $e^{y\log a+2n\pi i}$ (n=integer). It now follows from (1) that $e^{[\frac{1}{2}(22-17x)]\log 18+2n\pi i}=1$, whence $\frac{1}{2}(22-17x)\log 18+2n\pi i=0$, and therefore $x=\frac{22}{17}+(4n\pi i/17\log 18)$. When $n=0$, $x=\frac{22}{17}$. ED.

II. Solution by L. E. NEWCOMB; Los Gatos, California.

(1) Let $ABCD$ be the quadrilateral of area S^2, E the given point in the side AD; complete the triangle ABF and let $S_1^2=\frac{1}{2}S^2+$area DCF; $S_2^2=$areaABF $=S^2+DCF$; $GH=S_1$, $G'H'=S_2$. Draw ER and AR' perpendicular to BF, of lengths d and d', respectively. On FA lay off $FL=S_2$, $LN=S_1$, $FB'=FB$, $FA'=d'$, $FD'=d$; on FB lay off $FM=S_1$. Join LM and parallel to LM draw NO; join LO and parallel to LO draw $B'P$; join PD and parallel to PD' draw $A'T$; join ET. The line ET divides $ABCD$ equally.



In proof, $FL:FN=FM:FO=S_2:S_1=S_1:FO$, hence $S_2.FO=S_1^2$ and therefore $S_2^2.FO=S_1^2S_2$, which may be expressed as the proportion, $S_2^2:S_1^2=S_2:FO$. Now $LF(=S_2)$: $FO=B'F:FP$; therefore $S_2^2:S_1^2=B'F:FP$ from which follows easily, $S_2^2:\frac{1}{2}d'.B'F=S_1^2:\frac{1}{2}d'.FP$. But $B'F(=BF).\frac{1}{2}d'=S_2^2$, hence $\frac{1}{2}d'.FP=S_1^2$ and $FD'(=d):FA'(=d')=FP:FT$ and thus $\frac{1}{2}d.FT=\frac{1}{2}d'.FP$. Since $\frac{1}{2}d'.FP=S_1^2$ it follows that $\frac{1}{2}d.FT=S_1^2=\frac{1}{2}S^2+$ area DCF, and conseqently $\frac{1}{2}d.FT-$area $DCF=$area $ETCD=\frac{1}{2}S^2$.

(2) Let E' be the given points without the quadrilateral. For the construction of a line through E' cutting off a given area S_1^2 from the triangle ABF, see Geometry, Problem No. 218.

(3) Let E'' be the given point within the quadrilateral; draw $E''A''$ parallel to BF, EK perpendicular to AF and let $EK=a$. Extend AF, the distance $2a$, to F'. On the diameter $A''F'$ describe the semi-circle $A''K'F'$; draw FK' at right angles to AF; from the center K' with radius S_1 describe an arc cutting AF at L' and join $L'K'$. On FB lay off $FM'=a$, $FP'=S_1-L'F$; join NM', and parallel to to NM', draw $P'R''$. Through R'' and E'' draw $R''T'$ and let the altitude of the triangle $R''T'F$ to the base $R''F=b$. $R''T'$ is the required line.

In proof, $E''K(=a):b=R''A'':R''F$, hence $b.R''A''=a.R''F$; also $M'F$ $(=a):NF(=S_1)=FP'(=S_1-L'F):R''F$, hence $a.R''F=S_1.FP'=S_1.(S_1-L'F)$. Now $L'F=\sqrt{[S_1^2-(FK')^2]}$ and $(FK')^2=2a.A''E=2a.(R''F-R''A'')$, therefore $a.R''E=S_1(S_1-\sqrt{[S_1^2-2a(R''F-R''A'')]}$, consequently $\left(\frac{a.R''F}{S_1}-S_1\right)^2=S_1^2-$ $2a.R''F+2a.R''A''$, hence $\frac{a^2.(R''F)^2}{S_1^2}=2a.R''A''$, and $a.R''F=\frac{2R''A''.S_1^2}{R''F}$. But $a.R''E=b.R''F$, hence $b.R''A''=\frac{2R''A''.S_1^2}{R''F}$ and $b.R''F=2S_1^2$. $\therefore\frac{1}{2}b.R''F=S_1^2=$ $\frac{1}{2}S^2+$ area DCF. Consequentlp $\frac{1}{2}b.R''F-$area $DCF=\frac{1}{2}S^2=\frac{1}{2}$ of area of the quadrilateral $ABCD$.